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Modified Model for the Moon Sighting and Phases

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Research article

Modified Model for the Moon Sighting and Phases

Gamal Hafez

Keywords: Empirical Model, Moon Sighting, Crescent Visibility, Lunar Phases, Waxing crescent

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The modified moon sighting model is developed from the basic moon sighting model by adopting more phases. The preference of the new model is proved by its ability to predict extra three moon phases, first quarter, last quarter and dark moon. The modified model predicts the start date of 18 lunar months, from March 2022 to July 2023. The results of the new modified model correct the results of the basic model in 3 months, Dul-Hejjah 1443 H and Safar and Rabi-Second 1444 H. The comparisons of the new model 18 results are performed against the available results of different 3 references. The improved results of the new model are concluded by achieving the highest agreement rate of 79.2% with no single odd-result. The calendar of the modified moon sighting model is presented in a new innovative format.

1. INTRODUCTION

The basic moon sighting model of Hafez[1] was used to predict the crescent visibility and Hijri months' starts. The model was created and then tested for the months' periods, beginnings and full moon phases. But the model wasn't checked for the other moon phases, from the new crescent to the end of the month there exist about 28 phases. The ability of the model to also predict the moon phases is required as added value to the model to promote its application. But this feature is also important for the main purpose of the model, the crescent visibility. Every optimization in the calculations of moon phases will lead to higher accuracy in the crescent visibility and identifying the lunar months' starts. The observed moon phases from 2006 to 2020, by German mondphasen measurements which were published on,[2] will be used as reference values to develop the model and examine its accuracy.

Lunar Prospector (LP), NASA's third Discovery mission, was launched January 6, 1998 and ended on July 31, 1999. A lot of collected data was published by Konopliv et al.[3] They stated that the repeated moon's orbit around the earth is every 28 days and there exists another repeated orbit every three months. Also Lunar Orbiters were near equatorial with inclinations between 10◦ and 20◦. All discovery missions targeted the collection of every possible data about the moon motion, environment and structure. Later on Konopliv et al,[4] through the GRAIL Primary and Extended Mission, doubled the resolution of the gravity field with a fully dynamic least squares technique using spacecraft tracking data. But from the author point of view, exploring space in general and the moon in specific should be directed for scientific purposes and for more understanding of the space but not for military purposes or occupation of any part of the space. That is why the author insists on putting all space under agreement of keeping the space away from any conflict and treating it as a nature reserve.

The present work will focus on improving the accuracy of the basic moon sighting model.[1] First, the optimum start of sighting is selected by checking the effect of the shift angle with different start dates of sighting. Second, the ability of the sighting model to predict the moon phases is examined against the measurements of moon phases published in.[2] Third, According to the obtained results, the modified sighting model is developed. Fourth, the modified sighting model is applied and compared to the basic sighting model,[1] Umm-Alqura calendar,[5] visibility curves[6] and Al-Tawfiqat Alelhamih.[7] Noting that both 2 references [5] and [7] are considering the start date of any Hijri month is the next day of the crescent visibility of that month. But of course the crescent visibility date is the start of the first day and consequently the month's start. This will lead to that the month's start date in both [5] and [7] is one day more the exact crescent visibility date from their point of view. Fifth, a calendar of the modified sighting model is presented in a new developed form consistent with the nature of lunar months and natural (Islamic) days to produce one-calendar for the one Islamic Nation.

2. IDENTIFYING AN OPTIMUM START DATE OF THE MODEL

The basic moon sighting model BMSM of Hafez[1] can be written as

$$\frac{A}{D} = |\sin(wt + \theta)| \tag{1}$$

where:

A = the thickness of the luminous part of the moon.
D = the moon diameter.
w = the angular velocity of the moon, rad/day.
t = the time in days.
Θ = the shift angle from the sighting time 18:00:00.

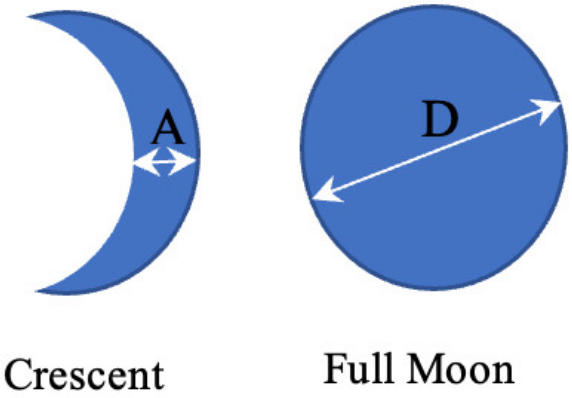

Fig. 1. Luminous Crescent.

The shift angle θ depends on the selected start date. Four start dates for the calculations of θ by the full moon condition (A/D=1) are selected from [2] and given in table 1. Using the information of full moon dates/times, 4 different modes of the sighting model are written:

$$A/D = |\sin(wt + \theta_i)| \quad i = 1, 2, 3 \; or \; 4$$

Using the 4 initial conditions of the 4 different full moon dates/times, θ1, θ2, θ3 and θ4 are calculated:

$$\theta_i = \frac{\pi}{2} - wt_i, \qquad i = 1,\ 2,\ 3 \; or \; 4 \tag{2}$$

The 4 sighting modes are used to calculate A/D of the full moon on 30-Dec-2020 at 04:28:18[2] and results are given in table 2.

The results are plotted in the figure 2 to visualize the dependence of the moon ratio A/D upon change of the start date:

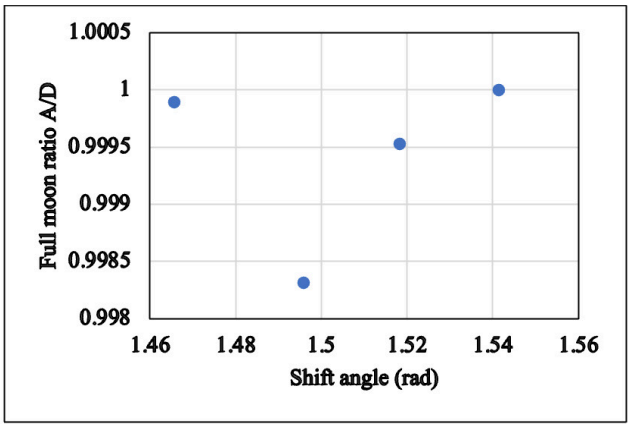

Fig. 2. The effect of the shift angle on the calculation of the full moon ratio of 30 Dec., 2020.

There are very small differences in the value of the full moon ratio. To confirm findings, another full moon is calculated by using the 4 modes. The measured full moon phase was on 21 May 2016 at 22:14:42 according to [2]. The obtained results are given in table 3 and are plotted in figure 3.

Both test cases give the same trend of the full moon ratios, in which θ3 of the full moon on 15 Mar-2016 gives

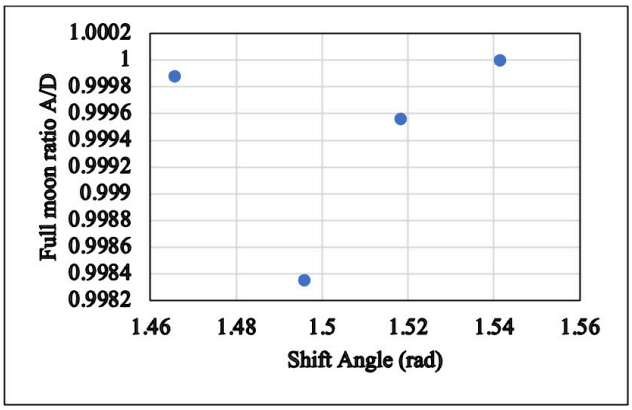

Fig. 3. The effect of the shift angle on the calculation of the full moon ratio of 21 May 2016.

the highest accuracy. On the other hand, the differences between the 4 calculations are practically negligible and the 4 different starts accurately predict the full moon as illustrated in figure 4.

Therefore, the basic moon sighting model is independent with respect to the start date or the shift angle as given in figure 4, from a sighting point of view. But from the accuracy point of view, the author preferred to continue the calculations in the rest of the present study by the shift angle θ3.

2.1. TEST THE BASIC MOON SIGHTING MODEL FOR THE MOON PHASES

Using BMSM with shift angle θ3 to calculate the moon ratios of quarter moon phases measured published on [2]. The dates and times of the quarter moon phases, both First Quarter FQ and Last Quarter LQ, of every next month to the initial date 14 March, 2006 are substituted into the BMSM. The obtained results of the moon ratios are recorded and compared to the expected actual value of A/D = 0.5. The calculated A/D of 58 different FQ and LQ phases are plotted in figure 5.

As can be seen in figure 5, the prediction of FQ phases or LQ phases are performing wave patterns. The FQ pattern is the lower wave. The LQ pattern is the upper wave. Both waves are obtained in higher values than the expected A/D of 0.5. These results confirm the need to improve the model to be able to accurately predict the phases of the moon.

3. THE MODIFIED MOON SIGHTING MODEL

The moon sighting model[1] proved its prediction for the new crescent visibility and hence the months' start dates. Adjusting the prediction of the moon phases is serving not only the moon phases themselves, but also the accuracy of the crescent visibility. The modified model is written in exponential form to keep the same sinusoidal behavior of the basic model.[1] The main form of the modified model is given by:

$$\frac{A}{D} = |\sin(wt + \theta)|^r \tag{3}$$

Table 1. the calculations of the shift angle θ

	Full Moon					t=0 at 18:00:00 one day before sighting			
i	**Month/year**	**Date**	**Hours**	**Minutes**	**Seconds**	**Add *6* Hours**	**days from every t=0**	**w rad/day**	**θ_i rad**
1	Jan-06	14-Jan	10	48	10	6	0.700115741	0.106383795	1.496315357
2	Feb-06	13-Feb	5	44	16	6	0.489074074	0.106383795	1.518766771
3	Mar-06	15-Mar	0	35	28	6	0.27462963	0.106383795	1.541580185
4	Apr-06	13-Apr	17	40	11	6	0.986238426	0.106383795	1.46587654

Table 2. The full moon ratio A/D by 4 different shift angles.

Sighting modes	Days from each t=0 up to 30-Dec-2020	BMSM
i		A/D of the full moon on 30-Dec-2020
1	5464.436319	0.998298029
2	5434.436319	0.999523195
3	5404.436319	0.999992878
4	5375.436319	0.999880388

Table 3. The full moon ratio A/D by 4 different shift angles.

Sighting modes		BMSM
i	Days from each t=0 up to 21-May-2016	A/D of the full moon on 21-May-2016
1	3781.176875	0.998341285
2	3751.176875	0.999545966
3	3721.176875	0.999995417
4	3692.176875	0.999868564

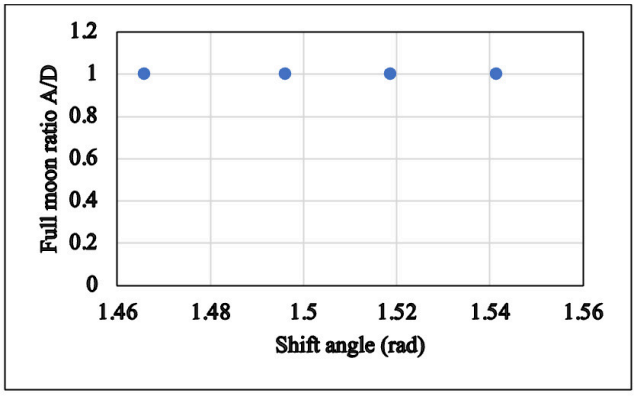

Fig. 4. The prediction of the full moon ratio 0f 30 Dec., 2020 by 4 different start dates.

Using the initial values, the start date of sighting on 14 March 2006 and the full moon on 15 March 2006,[2] the model can be written as:

$$\frac{A}{D} = |\sin(0.106383795051669 * t + 1.54158018456126)|^{r} \quad (4)$$

The exponent r is defined as Correction Exponent CE. To examine its influence in adapting the FQ and LQ phases, r is given four different values (1.75, 1.8, 1.85, and 1.9). Results of 9 quarter phases are given in figure 6.

Figure 6 shows the significant improvements of the predictions of quarter moon ratios A/D and how their values become closer to the exact value of 0.5. As a result, the use of the Correction Exponent (CE) r has been validated, but

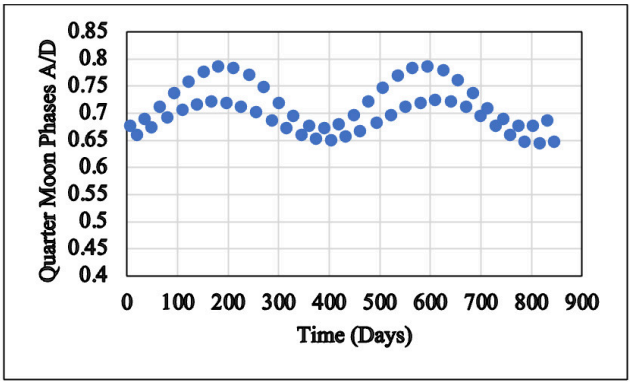

Fig. 5. Over prediction of A/D of the Quarter Moon Phases.

the required value of the exponent r can't be explicitly obtained by means of comparisons.

3.1. CALCULATIONS OF THE CORRECTION EXPONENTS

The Correction Exponent CE is calculated at each 58 quarter moon phases by substituting into Eq. (4) that A/D is equal to 0.5 and the respective time of each quarter. For example, the time t is substiuted by 21.7921875 days to calculaste r of a first-quarter moon phase FQ1 as follows:

$$\begin{aligned} \frac{A}{D} &= 0.5 \\ &= |\sin(0.1063838 * 21.7921875 \\ &\quad +1.54158018456126)|^{r_F} \\ \ln 0.5 &= r_F \ln|\sin 3.859915793| \\ (r_F)_1 &= 1.656806748 \end{aligned} \quad (5)$$

These calculations are applied to the 58 phases one by one. The resulting values of all 58 CEs are plotted into figure 7.

Figure 5 indicates a very important observation that the pattern of the Waxing moon phases differs from the pattern of the Waning moon phases. This observation implies to use two different CEs r, r_F for first quarter FQ and r_L for last quarter LQ. This observation is also illustrated in figure 7. Moreover, figure 7 shows that the obtained pattern of CEs in both FQs (lower pattern) and LQs (upper pattern) implies that rF and rL are wave functions and not constants. The general form of r according to its pattern in figure 7 is written as:

$$r = Constant + a\left|\sin\left(\frac{\pi}{T}t + \varphi\right)\right| \quad (6)$$

Where the constant c is a base value, a is an amplitude, T is a periodic time and φ is a shift angle to match between the formula and the r_F and r_L curves of FQ and LQ phases. For the rF curve, c is equal to 1.5, a is equal to 0.58 and T is equal to 413.4 days, see figure 7. To calculate the value of φ_F, the exact value of $(r_F)_1$ from Eq. 5 is used. Substituting into Eq. 6, the value of φ_F is obtained as follows:

$$\begin{aligned} (r_F)_1 &= 1.656806748 \\ &= 1.5 + 0.58\left|\sin\left(\frac{\pi}{413.4} * 21.7921875 + \varphi_F\right)\right| \\ \varphi_F &= 0.1081556761\ rad \end{aligned}$$

The final form of the CE model rF is obtained by introducing all parameters in Eq. (6) as follows:

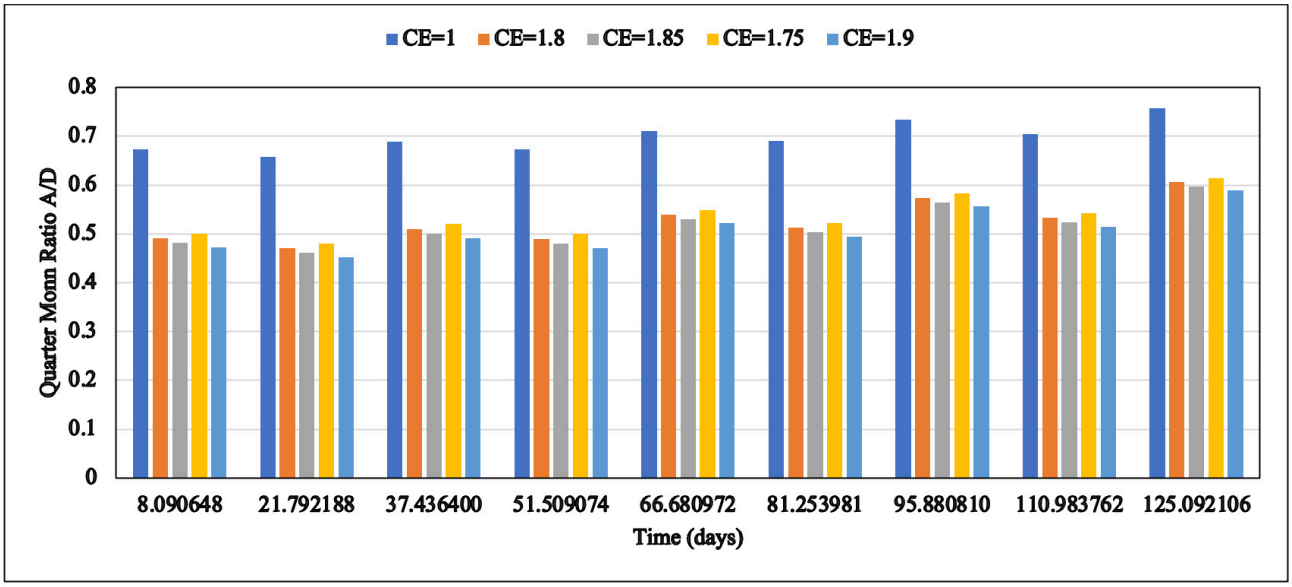

Fig. 6. The effect of Different Correction Exponents r on the Prediction of 8 Consecutive Quarter phases of the Moon.

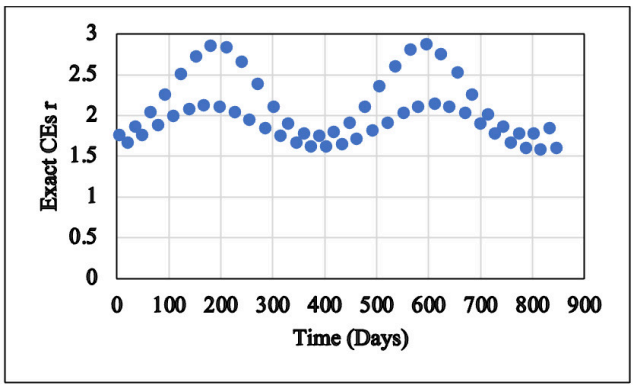

Fig. 7. The exact Correction Exponents calculated from the modified sighting model.

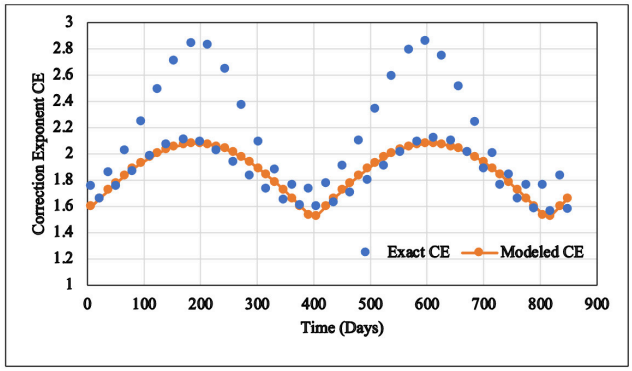

Fig. 8. The modeled CEs against the exact CEs of the First Quarters.

$$r_F = 1.5 + 0.58\left|\sin\left(\frac{\pi}{413.4} * t + 0.1081556761\right)\right| \quad (7)$$

The validity of the r_F formula of Eq. 7 is proved in figure 8 by plotting its graph against the graph of the exact r_F, figure 8.

Using the same technique, the CE formula of the LQs is obtained as given in Eq. 8 and then is validated in figure 9.

$$r_L = 1.7 + \left|\sin\left(\frac{\pi}{413.4} * t - 0.066068\right)\right| \quad (8)$$

Therefore the final form of the modified sighting model can be written as:

$$\frac{A}{D} = \left|\sin(0.106383795051669 * t + 1.54158018456126)\right|^{(r_F*\delta + r_L*(1-\delta))} \quad (9)$$

Where δ is the kronecker delta and equals 1 for the waxing moon phases (first half of the month) and equals zero at the second half of the month (waning moon phases).

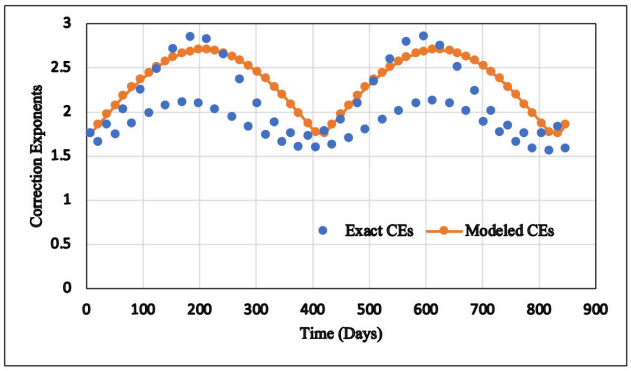

Fig. 9. The modeled CEs against the exact CEs of the Last Quarters.

The modified sighting model Eq. 9 is used to recalculate the 58 A/D of FQs and LQs to check the improvements. The obtained values are plotted in figure 10.

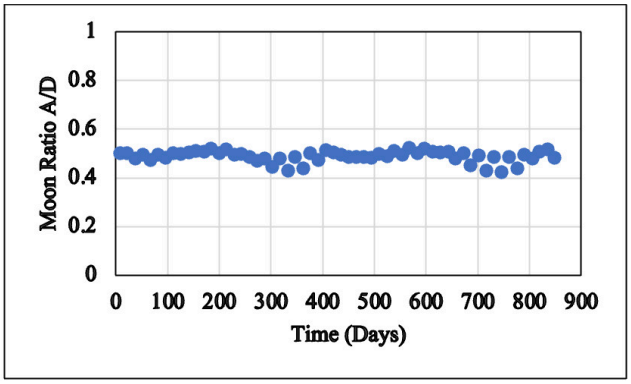

Fig. 10. Predictions of the modified sighting model fo r the quarter moon phases.

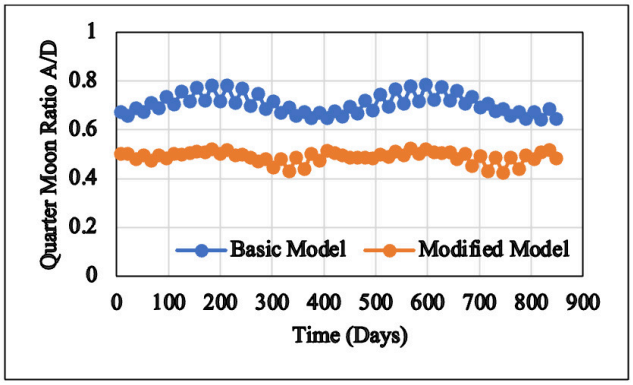

Fig. 11. Results of both Basic and Modified Models fo r Quarter Moon Phases.

Figure 10 confirms the accuracy of the modified sighting model to simulate the moon phases, as it predicts quarter moon ratios close to 0.5. This additional feature is important for more accuracy in the main purpose of the sighting model that is the new crescent visibility and Hijri months' start. Figure 11 compare the results of the modified moon sighting model with the basic moon sighting model. The modified sighting model eliminates the over prediction which causes the crescent enlargement. This advantage is cleared in the figure 12 by plotting the moon phases' calculations from both models. The pattern of the moon phases by the basic model is always higher (suffer from over prediction) than the pattern of the moon phases by the modified model. Even so, both models almost agreed about the crescent visibility and the full moon in many cases.

4. RESULTS OF THE MODIFIED MOON SIGHTING MODEL

The modified moon sighting model MMSM Eqs. [7-9] is applied to the period March 2022 to July 2023 to predict the start date of lunar months from Shaban 1443 H to Muharam 1445 H, 18 months. The basic moon sighting model BMSM results and comparisons for the same period was obtained in G Hafez.[1] The present calculations of MMSM differ from the calculations of BMSM just in 3 months out of 18 calculated months, namely: Dul-Hejjah 1443 H, Safar and Rabi-Second 1444 H. Three figures 13-15 compare the results of both models for these 3 months.

Figure 13 shows that the natural dark moon (A/D ≈ 0) is only obtained by the modified model on 28 and 29 June 2022. The first crescent of Dul-Hejjah 1443 H is found on 30 June 2022, as a correction to the BMSM prediction on 29 June 2022.

Figure 14 shows that, the natural dark moon is only obtained by the modified model on 26 and 27 August 2022. The first crescent of Safar 1444 H is found on 28 August 2022, as a correction to the BMSM prediction on 27 August 2022.

Figure 15 shows that, the natural dark moon is only obtained by the modified model on 25 October 2022. The first crescent of Rabi-Second 1444 H is found on 26 October 2022, as a correction to the BMSM prediction on 25 October 2022.

The results of the whole 18 months of the modified model and the four references [1,5–7] are regrouped in table 4. The discussions and conclusions are then presented in the next section.

5. DISCUSSIONS AND CONCLUSIONS

The MMSM corrects the BMSM in 3 months out of 18 months under the present study, they are highlighted orange in table 4. The comparison is done for the results of MMSM and the 3 references [5–7]. The summary of the comparison is highlighted in the first column of table 4 into 3 highlighted categories, namely green, blue and grey. The first category, the 8 months with the same crescent visibility date in all 4 techniques, is highlighted green. The second category, the 5 months with the same crescent visibility date in 3 techniques out of the 4 techniques, is highlighted blue. The odd techniques' results in these 5 cases of the second category are highlighted in red, see table 4. The third category, the 5 months with the same crescent visibility date in just 2 techniques out of the four techniques, is highlighted grey. Two techniques' results in these 5 cases of the third category are highlighted in yellow, see table 4.

The Advantages of The MMSM can be summarized in three main points. The accuracy is satisfied by the odd-free results, never defining the month start date as a singular date out of the 4 compared techniques, in other words never highlighted red. The MMSM is more natural as the crescent enlargement of the BMSM is eliminated, figure 12. Also the MMSM can predict the dark moon at the month's end whenever it wasn't predicted by the BMSM. The overall agreement of MMSM results through the comparison of the 4 techniques reaches the highest value of 79.2%. The MMSM is preferred than [6] which has the same agreement 79.2%, that is because in category 3 (highlighted in yellow) the results of [6] got the support from [5] which has the least agreement of 66.7% but MMSM has the support from [7] which has an average agreement rate of 71%. Therefore, the modified moon sighting model is considered accurate and natural enough to predict the start dates of lunar months

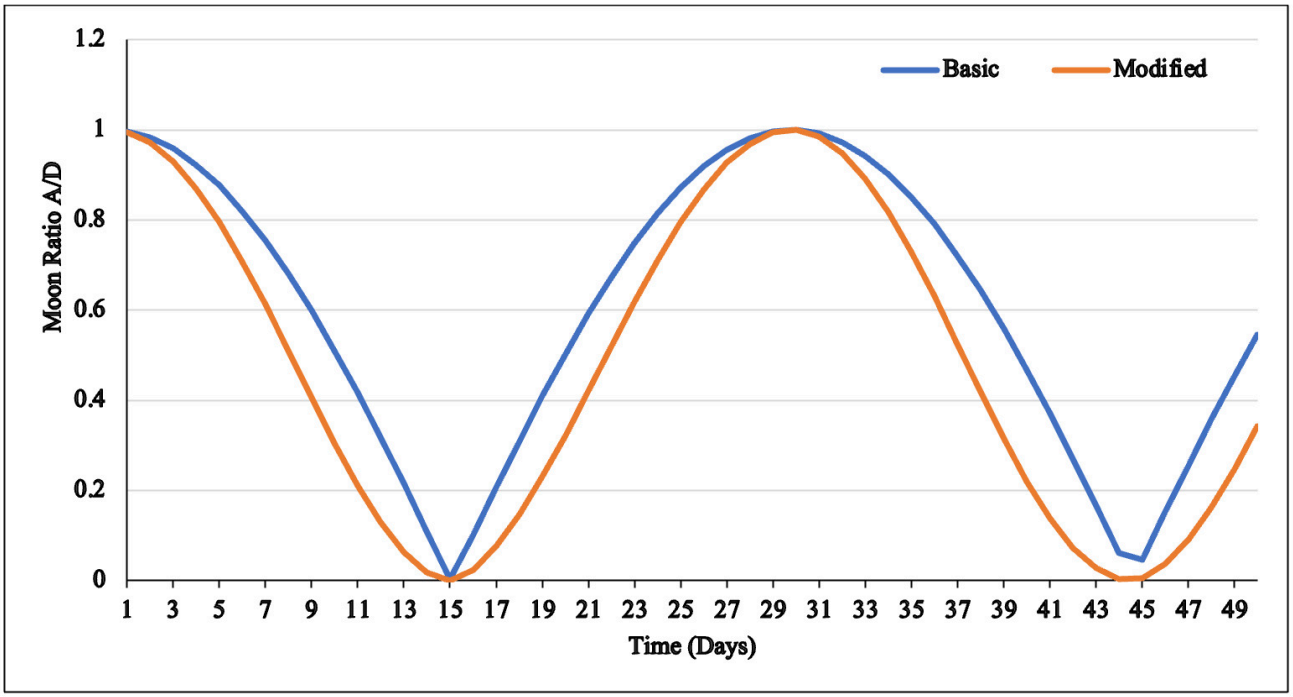

Fig. 12. The Comparison of the Moon Phases Calculations by both Basic and Modified Models.

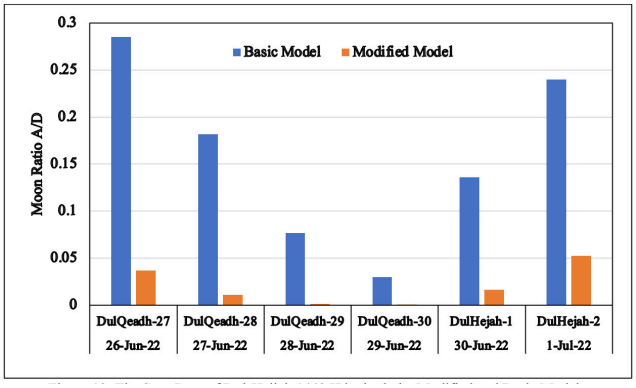

Fig. 13. The Start Date of Dul-Hejjah 1443 H by both the Modified and Basic Models.

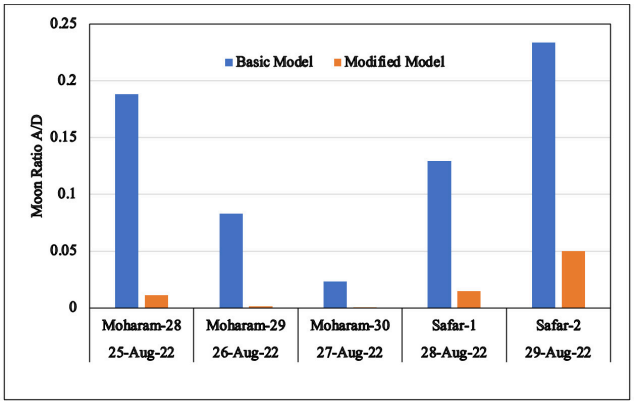

Fig. 14. The Start Date of Safar 1444 H by both the Modified and Basic Models.

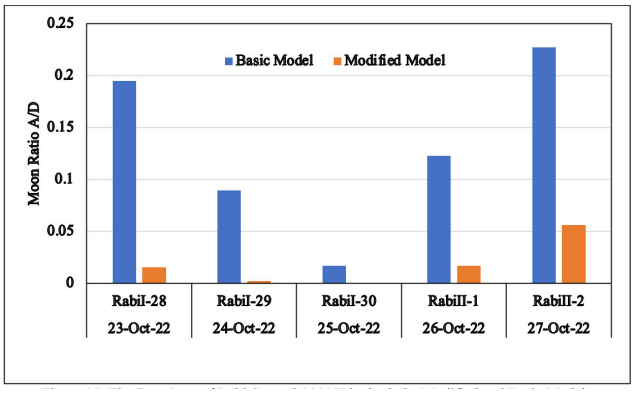

Fig. 15. The Start Date of Rabi-Second 1444 H by both the Modified and Basic Models.

with relatively better results than other techniques. The fruitful results of the modified moon sighting model is represented in a new developed calendar. The calendar contains both Gregorian and Hijri dates. The new design reflects the nature of both systems, Gregorian day's start is 12:00 am and Hijri day's start is sunset (Maghrib time), without any conflict. A sample of the resulting calendar is presented in figure 16.

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Countless thanks to ALLAH Almighty for His great care, grace, and guidance for me to complete this research, and I ask ALLAH to accept the present work and make it a beneficial work. I also thank my family for their continuous sup-

Table 4. The comparisons of crescent visibility dates/lunar months' start dates for 18 months.

Hijri Months	Basic Model[1]	Starting Date of Hijri Months (4 Techniques Under Comparison)			
		Um-Alqura Calendar[5*]	Visibility Curves[6]	Al-Tawqifat Alelhamih[7*]	Modified Model
Shaban-43	3-Mar-22	4-Mar	3-Mar	5-Mar	3-Mar-22
Ramadan	2-Apr	2-Apr	2-Apr	3-Apr	2-Apr
Shawal	1-May	2-May	1-May	3-May	1-May
Dul-Qaidah	31-May	31-May	31-May	1-Jun	31-May
Dul-Hejjah	29-Jun	30-Jun	29-Jun	1-Jul	30-Jun
Muharam-44	29-Jul	30-Jul	29-Jul	30-Jul	29-Jul
Safar	27-Aug	28-Aug	27-Aug	29-Aug	28-Aug
Rabi 1	26-Sep	27-Sep	26-Sep	27-Sep	26-Sep
Rabi 2	25-Oct	26-Oct	26-Oct	27-Oct	26-Oct
Jumada 1	24-Nov	25-Nov	24-Nov	25-Nov	24-Nov
Jumada 2	24-Dec	25-Dec	24-Dec	25-Dec	24-Dec
Rajab	22-Jan-23	23-Jan	22-Jan	23-Jan	22-Jan-23
Shaban	21-Feb	21-Feb	20-Feb	22-Feb	21-Feb
Ramadan	22-Mar	23-Mar	22-Mar	23-Mar	22-Mar
Shawal	21-Apr	21-Apr	20-Apr	22-Apr	21-Apr
Dul-Qaidah	20-May	21-May	20-May	21-May	20-May
Dul-Hejjah	19-Jun	19-Jun	18-Jun	20-Jun	19-Jun
Muharam-45	18-Jul	19-Jul	18-Jul	19-Jul	18-Jul

* The crescent visibility date = The month start date - 1 day.

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ABBREVIATION

BMSM Basic Moon Sighting Model
CE Correction Exponent
FQ First Quarter
LQ Last Quarter
MMSM Modified Moon Sighting Model

ليلة الأحد	نهار الاحد	ليلة الاثنين	نهار الاثنين	ليلة الثلاثاء	نهار الثلاثاء	ليلة الاربعاء	نهار الاربعاء	ليلة الخميس	نهار الخميس	ليلة الجمعة	نهار الجمعة	ليلة السبت	نهار السبت	1443 H
Sa-N	S-M	S-N	Mo-M	Mo-N	T-M	T-N	W-M	W-N	Th-M	Th-N	F-M	F-N	Sa-M	2022 G
2-Apr	3		4		5		6		7		8		9	
Ramadan-1		2		3		4		5		6		7		
	10		11		12		13		14		15		16	
8		9		10		11		12		13		14		
	17		18		19		20		21		22		23	
15		16		17		18		19		20		21		
	24		25		26		27		28		29		30	
22		23		24		25		26		27		28		
	1-May		2		3		4		5		6		7	
29		Shawal-1		***2***		***3***		4		5		6		
	8		9		10		11		12		13		14	
7		8		9		10		11		12		13		
	15		16		17		18		19		20		21	
14		15		16		17		18		19		20		
	22		23		24		25		26		27		28	
21		22		23		24		25		26		27		
	29		30		31		1-Jun		2		3		4	
28		29		30		Dul-Qaidah-1		2		3		4		
	5		6		7		8		9		10		11	
5		6		7		8		9		10		11		
	12		13		14		15		16		17		18	
12		13		14		15		16		17		18		
	19		20		21		22		23		24		25	
19		20		21		22		23		24		25		
	26		27		28		29		30		1-Jul		2	
26		27		28		29		30		Dul-Hejah-1		2		
	3		4		5		6		7		8		9	
3		4		5		6		7		8		****9****		
	10		11		12		13		14		15		16	
****10****		***11***		**12**		*13*		14		15		16		
	17		18		19		20		21		22		23	
17		18		19		20		21		22		23		
	24		25		26		27		28		29		30	
24		25		26		27		28		29		Moharam-1-1444		

Fig. 16. The developed form of the calendar taking into account the natural differences between Lunar and Gregorian months. (First row Contains Days Names in Arabic. Every Day is divided into 2 Cells to represent the Night and Morning. Second Row contains Days in English. The first Cell is Sa-N which represents Saturday Night. The last Cell is Sa-M which represents Saturday Morning. So always M stands for Morning and N stands for Night).

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