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THE DISTORTION ENERGY THEORYANALYSIS OF SPUR GEARS

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ABSTRACT

Gears play a major role in power transmission. A precise knowledge of the stresses on gear tooth during operation is very essential in designing a reliable gear transmission system. This paper analyses the stresses on gear tooth, at both tension and compression sides, of the tooth. The analysis of the tooth strength in this paper is based on the distortion energy theory, (Von Mises Theory). A new equation similar to Lewis's equation was obtained but with a different form factor. The newly derived equation is expected to obtain smaller and hence more economical gears for the same transmitted torque.

Keywords: *Spur gears, Gear strength, Distortion energy theory, Lewis's equation, Form factor.*

1. INTRODUCTION

One of the most important elements in power transmission is gears. A lot of research works have been dedicated to improve the design and manufacturing of gears. Tooth strength and contact stress on tooth surface are the major factors in designing gears [1, 2]. In order to improve gear design, other factors such as load dynamic effect or shocks, tooth size effects, and load distribution are also introduced [3 and 4]. As a result of the torque transmitted, the load acting on a single pair of spur gear is shown in Figure 1. The normal force, (F_n), transmitted from one tooth to another is normal to the contact surfaces and is inclined to the common tangent, of the two gears, by an angle ϕ, (pressure angle).

The normal force can be analysed into two components:

Tangential force, $F_t = F_n \times \cos\varphi$

Radial force, $F_r = F_n \times \sin\varphi$

$$F_r = F_t \times \tan\varphi \qquad (1)$$

φ *:* Pressure angle

These are the major forces in view of gear teeth strength.

2. LEWIS EQUATION FOR GEAR STRENGTH

W. Lewis was the first to develop a useful equation to estimate the bending stress in a gear tooth in 1892. The equation is based on stress resulting from the tangential component (F_t) only. The equation ignores the radial component (F_r) completely. The tooth is considered as cantilever with contact taking place at the tip of the tooth. The tangential component (F_t), only, is considered in the analysis, see Figure 1. Also Lewis assumed that only one pair of teeth is in contact at any time. The effect of the fillet, at the tooth root, on the stress concentration is ignored. The normal force is assumed to be uniformly distributed along the face width. For the cantilever formed tooth, the maximum stress is found at the tooth root.

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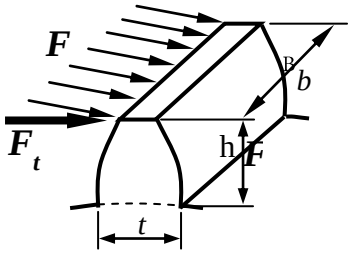

Fig.1. Load acting on a single pair of spur gear

Applying the standard bending stress equation ($S=\frac{M}{Z}$) leads to:

$$S=\frac{6F_t h}{bt^2} \quad (2)$$

Where,

S = bending stress in gear tooth

F_t = tangential tooth load

b = face width of tooth

h = tooth depth

t = tooth thickness

The factor, $\frac{t^2}{6h}$ the geometrical property of the size and shape of the tooth, is expressed as a function of the circular pitch p_c by the relation:

$$\frac{t^2}{6h}=yp_c \quad (3)$$

where y is known as the Lewis form factor.

Replacing the diametral pitch instead of the circular pitch, the form factor is denoted by Y, ($Y = \pi y$), the factor Y is also called the form factor.

Also $t^2=6hyp_c=\frac{6hYp_c}{\pi}=\frac{6hY}{P_d} \quad (4)$

Substituting equation (4) with equation (2) we obtain:

$$S_b=\frac{F_t P_d}{bY} \quad (5)$$

The Lewis form factor depends on both the number of teeth N and the pressure angle (ϕ). It is available in a form of a chart or a table.

3. AGMA GEAR STRESS EQUATION

AGMA developed an equation based on Lewis's equation. AGMA equation it takes into account:

1. Stress concentrations at the tooth root
2. The effects of multiple pairs of teeth in contact

$\sigma = W_t P K_v K_o K_m K_s K_B / FJ$

J : gear geometry factor

K_v : velocity factor

K_O : overload factor

Km : mounting factor

Ks : size factor

K_B : rim thickness factor

Many techniques have been employed in order to improve gear performance. Figure 2 shows the analysis of two gear teeth in contact utilizing a photo-elasticity technique. A bending a tensile stress is generated at A and a compressive stress at B. The radial component of the tooth force (F_r) increases the compressive stress at B. The rolling and sliding of two teeth surfaces, at point C, develops a contact stress situation which may result in a surface fatigue of the tooth [6].

FEM is applied for complicated geometry in order to obtain the stress of an elastic body. There have been numerous research studies in this area, See Figure 3 [5, 6, 7].

Raptis et al compared the results of photo-elasticity experiments and the results of the FEM obtained by ANSYS software. They concluded that the deviations between the two methods are acceptable. This is because of the possible errors that may occur during the execution of the two methods [3].

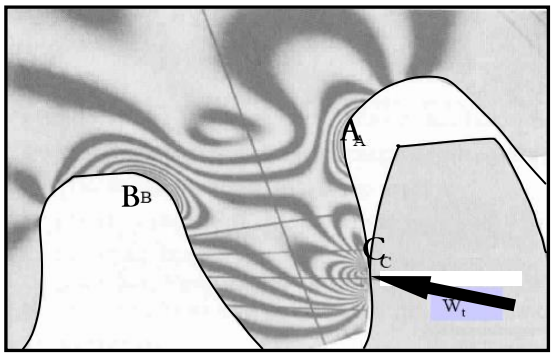

Fig.2. Photo-elastic analysis of spur gear teeth

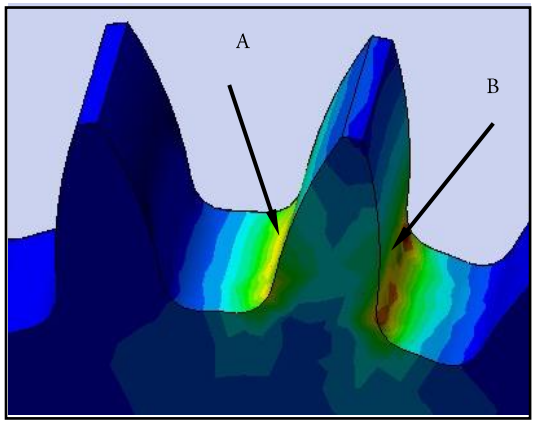

Fig.3. Finite element analysis of gear stress

The effect of stress concentration, on the gear tooth failure, was investigated by Sankar et al. In order to avoid tooth damage they introduced profile modification in root fillet. A circular root fillet instead of the standard trochoidal root fillet is introduced in spur gear and analyzed using ANSYS software. [8]

Hassan [9], applied FEM to study the surface failure resulting from repetitions of the contact stress, by applying finite element method. He reported that stress results obtained from both the AGMA calculation method and that obtained by the method he applied are practically identical.

This work applies the distortion energy theory because, based on experiments failure in the case of ductile materials is considered to be due to shear stress.

4. DISTORTION ENERGY THEORY ANALYSIS

Considering the effect of the normal force acting on a single tooth there are three important stresses that will be given extra attention in this analysis. Other stresses like contact stresses will be ignored as the analysis is only concerned about the stresses at the gear tooth root.

The main stresses acting at the root are:

1. Bending stress (S_b) resulting from the tangential force F_t.
2. Compressive stress (S_c) resulting from the radial force F_r
3. Shear Stress (S_s) resulting from the tangential force F_t

The bending stress is as shown by equation (5), Lewis's modified equation:

$$S_b = \frac{F_t P_d}{bY} \tag{6}$$

The compressive stress due to the radial load, F_r can be shown equal to:

$$S_c = \frac{F_r}{bt}$$

$$S_c = \frac{F_t \times \tan\varphi}{bt} = \frac{F_t \tan\varphi}{b\sqrt{\frac{6hY}{P_d}}}$$

But the tooth depth, $h = \frac{2.157}{P_d}$, [10]

$$S_c = \frac{F_t \tan\varphi}{b\sqrt{\frac{6 \times 2.157 \times Y}{P_d \times P_d}}}$$

$$S_c = \frac{0.28 \times F_t \tan\varphi \times P_d}{bY^{0.5}} \qquad S_c = 0.28\frac{F_t P_d}{bY} Y^{0.5} \tan\varphi \tag{7}$$

The total axial stress:

$$S_x = S_b \pm S_c$$

$$S_x = \frac{F_t P_d}{bY}\left(1 \pm 0.28 Y^{0.5} \tan\varphi\right) \tag{8}$$

Similarly the shear stress due to the tangential load F_t can be shown equal to

$$S_s = \frac{F_t}{bt}$$

$$S_s = 0.28\frac{F_t P_d}{bY} Y^{0.5} \tag{9}$$

The distortion energy theory, gives the following equation for the condition of failure for a combination of a direct tensile stress S_x and a shear stress S[10,11].

$$2S_y^2 = (S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2 \tag{10}$$

In the case of combined load the member will fail when its Distortion Energy Theory (also known as Von Mises Theory) exceeds the distortion of the tensile elastic limit, in other

words for a direct stress S_x and a shear stress S_s :

$$S_e^2 = S_x^2 + 3S_s^2 \quad (11)$$

S_e: elastic limit stress

Substituting equation 8 and 9 with equation 11

$$S_e^2 = \left[\frac{F_t P_d}{bY}\left(1 \pm 0.28 Y^{0.5}\tan\varphi\right) \right]^2 + 3\times\left[0.28\frac{F_t P_d}{bY} Y^{0.5} \right]^2 \quad (12)$$

$$S_e^2 = \left[\frac{F_t P_d}{bY} \right]^2 \left[\left(1 \pm 0.28 Y^{0.5}\tan\varphi\right)^2 + 0.2352Y \right]$$

For a pressure angle of $20^0, \tan\varphi = 0.364$

$$S_e^2 = \left[\frac{F_t P_d}{bY} \right]^2 \left[\left(1 \pm 0.102 Y^{0.5}\right)^2 + 0.2352Y \right]$$

$$S_e = \frac{F_t P_d}{bY}\sqrt{\left(1 \pm 0.102 Y^{0.5}\right)^2 + 0.2352Y}$$

$$S_e = \frac{F_t P_d}{bY}\sqrt{\left(1 \pm 0.102 Y^{0.5}\right)^2 + 0.2352Y}$$

$$F_t = \frac{S_e bY}{P_d}\frac{1}{\sqrt{\left(1 \pm 0.102 Y^{0.5}\right)^2 + 0.2352Y}}$$

$$F_t = \frac{S_e b}{P_d}\frac{Y}{\sqrt{\left(1 \pm 0.102 Y^{0.5}\right)^2 + 0.2352Y}} \quad (13)$$

Equation 10 can be written as:

$$F_t = \frac{S_e b Y_n}{P_d}$$

where $Y_n = \dfrac{Y}{\sqrt{\left(1 \pm 0.102 Y^{0.5}\right)^2 + 0.2352Y}}$

Y_n is the new form factor taking into account the effect of the combined stresses and applying the Distortion Energy Theory. The factor can be simplified as:

$$Y_n = \frac{Y}{\sqrt{1 \pm 0.204 Y^{0.5} + 0.246Y}} \quad (14)$$

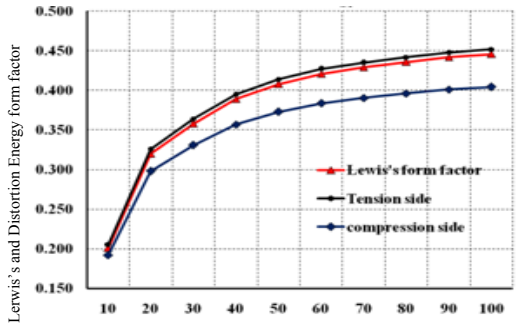

Fig.4. Comparison between Lewis's equation and The Distortion Energy Theory equation

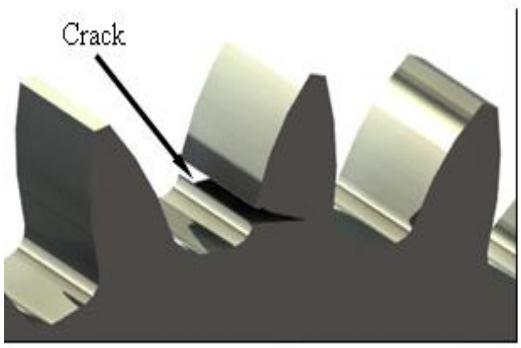

Fig.5. Crack propagation

5. CONCLUSION

A new formula for the form and stress factor is obtained and it takes into consideration the tangential stress, compressive stress as well as the shear stress. Figure 4 shows Lewis's plot and the distortion energy theory form factor. The distortion energy theory form factor is the plot for both tension and compression sides of the gear tooth.

TENSION SIDE:

The new form factor for the tension side is slightly higher than Lewis's form factor. This indicates that using the new form factor will result in smaller economical gear design. This is a result of including the effect of the radial compressive component (Fr) of the driving force (Fn).

COMPRESSION SIDE:

On the other hand Figure 4 shows a lower form factor at the compression side. This is due to the effect of the radial component increasing the total stress in the compressive side. Gear-tooth failure due to bending fatigue is generally the outcome of the crack originating in the root of the tooth and failure usually occurs on the tension side as a result of crack initiation and then propagation towards the compressive side (Figure 5). The tooth is capable of transmitting lower force.

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تحليل الاجهادات على التروس المهمازية بتطبيق نظرية طاقة التشوه

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الملخص:

تلعب التروس و العجلات المسننة دوراً رئيسياً في نقل القدرة بين الأعمدة. إن المعلومات الدقيقة عن الاجهادات التي تؤثر على اسنان الترس خلال التشغيل مهمة عند تصميم مجموعة تروس تتميز بالاعتمادية. هذا البحث يحلل الاجهادات التي تؤثر على اسنان التروس في جانبى السنة. يعتمد تحليل الاجهاد في هذا البحث على نظرية طاقة التشوه (نظرية فون مايسيس). تم التوصل إلى معادلة مشابهة لمعادلة لويس الشائعة و لكن بمعامل شكل جديد. يتوقع أن ينتج من استعمال المعادلة الجديدة تروس أصغرقليلاً من تلك التي تنتج من معادلة لويس ومن ثم تروس أصغر لنقل نفس عزم الدوران.