Yanbu Journal of Engineering and Science

Volume 7 | Issue 1 Article 8

2013

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Recommended Citation
El-Behairy, Fawzy M. A. (2013), LOCALIZATION PHENOMINA AND FRACTURE ENERGY OF CONCRETE AT EARLY AGES, *Yanbu Journal of Engineering and Science*: Vol. 7: Iss. 1, 65-74.
DOI: https://doi.org/10.53370/001c.24178
Available at: https://yjes.researchcommons.org/yjes/vol7/iss1/8

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Yanbu Journal
of
Engineering
and Science
ISSN: 1658-5321

Vol. 7, October 2013 (1434H)
www.yjes.org.sa

LOCALIZATION PHENOMINA AND FRACTURE ENERGY OF CONCRETE AT EARLY AGES

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ABSTRACT

Several experimental studies have been done long time ago to calculate the fracture energy of concrete [2, 6]. Most of these studies neglected the loss of energy due to the frictional process in the mechanism of experimental set up. All these studies have used the indirect tensile testing method, as a suitable approach for determining the fracture energy of concrete of mature age [3, 7]. However all these experimental studies have not investigated the crack or the localization process of young age concrete yet.

So, to study the fracture and localization process at early ages of concrete, this experimental investigation study is done. Fracture energy for concrete at early ages is estimated experimentally considering the weather of Kingdom of Saudi Arabia. The specimens at early age are tested to calculate the fracture energy of concrete using a direct tension test, taking into consideration the loss of energy due to the frictional process in the mechanism of experimental set up. Also the crack localization is considered during the experiment, as dealing with the localized fracture. In this experimental work, the measurements reach to the post peak softening branch of concrete using the direct tension. The results show that there is a fairly good agreement between the experimental results and the numerical output using the Modified Lattice Model [9,10,11].

Keywords: *Fracture energy, Localization, Early Age, Softening, Modified Lattice Model.*

1. INTRODUCTION

1.1 GENERAL

The problem of fracture in massive concrete structures at early age during or immediately after construction is a major design concern to practicing engineers. Such fractures are primarily caused by the differential temperature gradient due to the heat generated by the process of hydration. Although there are code provisions (ACI Committee 207- 2000, ACI Committee 318-2003) to cope up with such cracks [4, 5], the actual mechanism of such cracking and strain localization in concrete is still a matter of research. The rapid increase in the number of such large scale structures such as dams of hydroelectric power plants, basement concrete of nuclear power plants, large pavements, bridge piers etc. In the design of plain or reinforced concrete structures, cracking is taken into account in predicting the behavior in service conditions since the length, width and spacing of cracks have major influence on structural behavior as well as on its long term durability. Since in large scale structures, cracks tend to form from the very beginning of construction, it is necessary to understand the fracture characteristics of concrete at early ages.

Received: May 26, 2012 Accepted: October 20, 2012

All the previous works have a similarity that the analysis was done in the framework of classical continuum plasticity or damage modeling and most of them neglected fracture energy or the postulates of fracture mechanics [20]. We are still lacking an adequate numerical model or method to successfully describe such a complex process.

In this research, we focus our attention on the computational mechanics of early age concrete, which is partly plastic and partly brittle frictional material. Failure of such material is characterized by highly non-linear stress-strain and volume change relations and its failure usually involves strain localization in the form of cracks or shear bands. The main objective of the present study is thus the clarification of the fracture and localization problems of concrete at early age through finite element application. In this study the Modified Lattice Model is implemented as a numerical analysis.

2. EXPERIMENTAL INVESTIGATION OF CONCRETE FRACTURE AT EARLY AGES

2.1 INTRODUCTION

In this experiment the effect of lateral loads on several RC elements are done. The loading was continuously carried out until the failure of the RC element, calculate the area under the experimental load vs. displacement curve to get the total fracture energy and then divide this quantity by the cross sectional area at the notch to get the unit fracture energy G_f. One of the disadvantages of this method, is that the fracture area is relatively small compared to the overall volume which results in a somewhat overestimated value of fracture energy.

During this test, the experiment must be carried out at a slow loading rate to avoid any dynamic effect and the effect of dead load

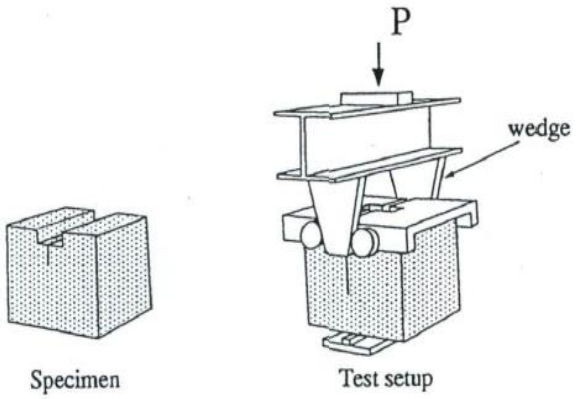

Fig.1. Wedge splitting test

must be taken into account in the process of calculation.

This method is suitable for hardened concrete only. Because at early age, concrete is too soft to hold its own weight if any beam is made. Another method is the wedge splitting test. It was first proposed by Tschegg and Linsbauer (2006) which seems to be very effective in determination of the fracture energy as in Fig.1. The advantage of wedge splitting test over three point bending test is that its crack area is large compared to the volume of the specimen and it is suitable for high rate loading. Dead weight also has no influence on the fracture energy because in this method, fracture energy is calculated on the basis of the horizontal component of the force and the crack opening displacement.

2.2 TEST SETUP

This section describes the test specimens and test set up machines. Mix and casting procedures are also described. These tests are performed in the strength and concrete laboratory.

2.2.1 Testing Machines

A typical setup of the testing apparatus is shown in Fig.2. Using ordinary testing machines, it is quite difficult to get the post peak softening branch of concrete in direct tension test. For this reason, two different types of testing machines were used

(Lokuliyana and Tanabe 2002) [7,13]. One for concrete at early ages and the other one for later ages of concrete. For strengths equal to or more that 0.5 MPa or ages over 1 lhr a high stiffness testing machine was used (Fig.2.). Due to the high stiffness loading columns, this machine was able to get the softening branch of both tension and compression with high accuracy.

The minimum loading rate that can be used in this Machine is $2.0x10^{-4}$ mm and this is controlled by a computer. The maximum tensile loading capacity of this machine was 150 kN. The maximum and minimum compressive loading capacity was 1500 kN and 1.0 kN respectively. The other testing machine which was used to test specimens at early age (ageless that ll hrs or strength less than 0.5MPa) had the maximum and minimum loading capacities of 50 kN and 0.031 kN respectively.

For this machine, the loading rate was approximately 0.18 MPa per minute. The strain was controlled manually during loading. The loading speed of tension testing was 0.1 MPa to 0.3 MPa. The time from loading to the failure varied from a few minutes to over 10 minutes according to the strength of the specimen and loading rate.

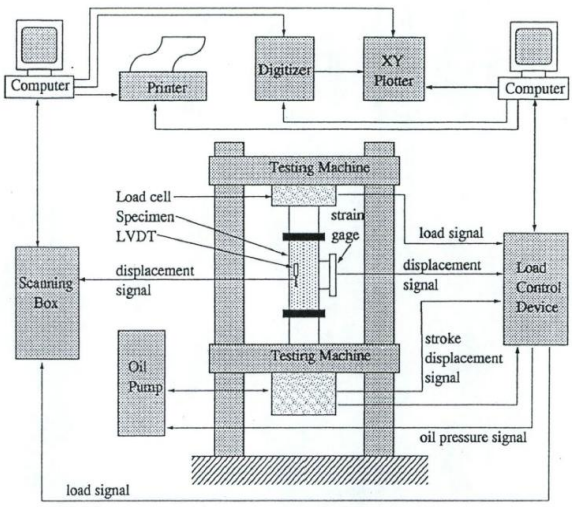

Fig. 2. Experimental setup of the high stiffness testing apparatus

2.2.2 Specimens

The size of the specimens used in the series of test was rectangular prisms having dimension of 300mm x 140mm x 40mm. To facilitate loading, each end of specimens had two pieces of 18 mm diameter steel rods embedded. The length of the rods was about 100 mm to ensure proper bond as shown in Fig. 2.3. The specimens were cast in steel molds having a pair of plastic plates to form notch at the middle of the specimen. On the front face of the specimen six pairs of steel studs were placed for the attachment of strain gauges. Displacement transducer type strain gages were used to measure strains across the fracture process zone. The distance between the two studs in a pair, i.e. the gage length, was taken as 20 mm, 30 mm and 40 mm. Another two sets of strain gages were affixed on the two sides of the specimen with a gage length of 50 mm to measure vertical strain at the sides as shown in Fig.4.

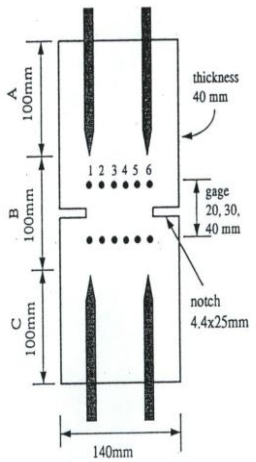

Fig.3. Direct Specimen Details

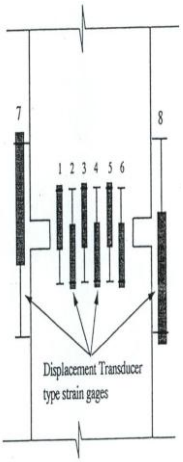

Fig.4. Details of Tension the Location of Strain Gages

2.2.3 Concrete Mix and Casting of Specimens

The proportion of the different constituents of the concrete mix is listed in Table 1 Coarse aggregate was crushed stone of maximum aggregate size 13mm. It came from the

Yoshino river of Tokushima Prefecture. Fine aggregate was the river sand from the same river with a maximum aggregate size of 5mm. It had fineness modulus of 2.50. The specific gravity of the aggregate and cement was 2.60 and 3.15 respectively (Lokuliyana and Tanabe 2002).

Concrete was mixed manually and poured in to the mold and compacted by a vibrator having vibration frequency of 220 Hz to 250 Hz. The vibrator was applied for one to three seconds. The mixing temperature was 30 °C. Time was measured from the moment water was added to the mix. The mold was placed horizontally during casting. Specimens tested at very early ages were removed from molds just before testing. Others specimens were removed from molds after 24 hours and were placed in water bath at a temperature of 20 °C.

TABLE 1: CONCRETE MIX PROPORTIONS

Air *(%)*	W/C Ratio	Mix Quantities (kg/m)				Slump (cm)
		Water	Cement	Fine Agg.	Coarse Agg-	
2.5	0.55	185	336	840	1008	8

2.2.4 Preliminary Testing

In order to get some idea on the feasibility of the testing procedure some preliminary testing with different notch configuration was made (Lokuliyana and Tanabe 1992). For this purpose specimens were tested without notch at the age of 15 hrs. Another two sets of specimens were also tested at the same age but with notches of 1.4 mm and 4.4 mm width. The notches were made by diamond saw cutting machine. For un-notched specimens, the crack formed not at the center but at a distance of approximately 90 mm from the end. At this location exists the ends of the embedded steel bars. So, the failure at this position can be considered due to the weaker concrete and imperfection caused by the presence of the steel bars. In the case of 1.4 mm wide notch, crack still formed at a similar place before. But for the notch of 4.4 mm width crack formed at the central position through the notch. In this test, electrical wire gage type strain gages were placed on the front face of the specimen and during testing most of the gages were removed by the failing concrete. This was another difficulty faced in placement of the gages in early age concrete. This difficulty was overcome by using LVDT type strain gages which were fixed to specimens using small studs as shown in Fig.4. and Fig.5.

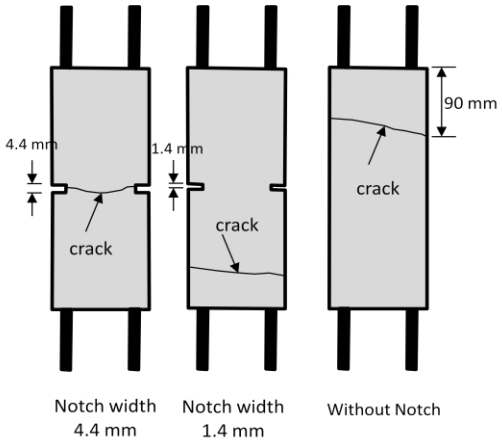

Fig.5. Results of preliminary tests

2.2.5 Testing Specimens at Early Ages

Tests on concrete specimens were made at different ages. Due to the difficulties in preparing and fixing the specimens in the testing machines, the time interval between ages was not regular. Tests on concrete specimens were performed at ages of 8, 9, 12, 16, 18, 20, 22, 24, 43, 48, 72, 80, 96, 102, 104 and 317 hrs after casting of specimens. The effect of gage length was studied by varying the gage lengths. The different gage lengths that were used were 20, 30 and 40 mm. Out of the set of the test results, the results of a test specimen tested at the age of 20 hours with gage length of 20mm is selected and the variation of strain along the crack path for peak load and at failure is shown in Fig.6.

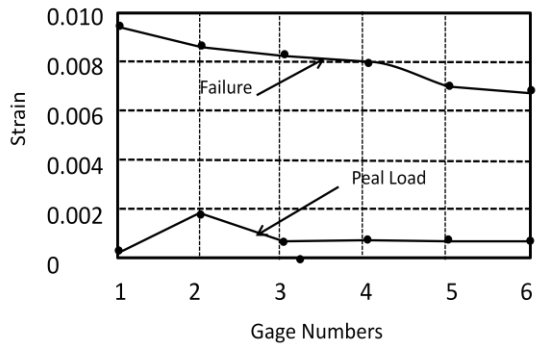

Fig.6. Strain distributions for a 20 hour specimen

2.3 DETERMINATION OF FRACTURE ENERGY

The area under the stress-strain diagram gives the fracture energy per unit volume of the fracturing material in the fracture process zone. However, in fracture mechanics the definition of fracture energy is defined in a different way. Fracture energy, G_f , is defined as the amount of energy required to produce a cracked surface of unit area. Thus if we multiply the width of the fracture process with the area of the stress-strain diagram then we obtain the fracture energy G_f thus

$$G_f = w\int_0^{\infty} \sigma d\varepsilon \qquad (1)$$

TABLE 2: SUMMARY OF EXPERIMENTAL RESULTS

Age (hours)	Young's modulus (MPa)	Tensile strength (MPa)	Fracture Energy G_f N/mm
9	400	0.14	8
16	1000	0.25	24
24	6000	0.80	28
48	8000	1.00	36
72	11000	1.50	54
102	14000	2.20	84

Where ω is the width of the fracture process zone, σ is the stress normal to the crack surface and ε is the strain normal to crack surface. For the experimental determination of the fracture energy, we make use of Eq. 1. We use the stress strain data obtained from experiments and consider the gage length as the width of the fracture process zone. Using the above procedure, fracture energy was calculated for different ages of concrete based on the experimentally obtained stress-strain relations. Since a total of eight strain gages were used per specimen and also there were different gage lengths, an appropriate averaging was made in determining the fracture energy. Other physical properties of concrete such as Young's modulus, tensile strength etc. was also determined from the experimental results. A summary of the experimentally obtained results are presented in Table 2.

Based on the experiments, Lokuliyana and Tanabe (2002) made a regression analysis to obtain an empirical equation relating the fracture energy with the other physical properties of concrete [16]. In doing so, they assumed a linear relationship between the characteristic length l_{ch} and tensile strength. Through the regression analysis they obtained the following equation of fracture energy as in Eq. 2.

$$G_f = (1.0 - 1.07 f_t)\frac{f_t{}^2 da}{E} \qquad (2)$$

Where f_t is the tensile strength (MPa), da is the maximum aggregate size (mm), E is the Young's modulus (MPa) and G_f is the fracture energy in N/mm. It should be noted that the assumption of linear variation of characteristic length with the change of tensile strength may not be a good choice.

3. NUMERICAL ANALYSIS FOR THE TEST RESULTS

3.1 INTRODUCTION

At early age, concrete undergoes rapid chemical and physical changes. Shrinkage and creep also starts taking place. In this research, we are interested only in the mechanical behavior of the early age concrete under the application of load. So we will not consider the possibility of cracking due to the shrinkage or rate dependent creep.

3.2 Numerical Finite Element Analysis

For the purpose of numerical analysis, the Modified Lattice Model in three dimensions which is produced before by the author [9,10,11] is applied. The original specimen was 300 mm long with a pair of 18 mm diameter steel rods embedded longitudinally at each end of the specimen. The length of the embedded rods was 100 mm. Due to the presence of these rods the outer 100 mm portions at both end of the specimen were rather stiff compared to the central 100 mm portion. This is especially true when we are dealing with young age concrete. At early age both the stiffness and the strength of concrete are very low.

TABLE 3: VALUES OF MATERIAL PARAMETERS

Age	9 hrs	16 hrs	24 hrs	48 hrs	72 hrs	102 hrs
E, MPa	400	1000	6000	8000	11000	14000
V	0.15	0.15	0.15	0.15	0.15	0.15
C_1	1.0	1.1	1.0	1.0	3.0	1.0
C_2	5.0	5.5	4.0	5.0	5.0	5.0
K_u	0.009	0.011	0.01	0.009	0.01	0.012
f_t, MPa	0.14	0.25	0.8	1.0	1.25	2.2
l, mm	1.6	2.0	1.0	1.1	1.1	0.8

So it is quite reasonable to assume that for young age concrete, the two ends having embedded rod elements will behave almost rigidly as compared to the central portion.

3.3 Results and Discussions

The results of the finite element analysis are shown in from Fig.7. through Fig.8. using the Modified Lattice Model [9,10,11]. These figures show the comparative aspect of the numerically obtained stress-strain diagram with the experimental one. Actually there are six strain gages in one specimen and numerically obtained data was tried to fit with all the corresponding experimental results. But due to space limitations only one diagram is shown here for each age of concrete. It should be noted that the experimentally obtained stress strain diagrams for all the strain gages of all specimens were not completely satisfactory. Because it is very difficult to handle the concrete at early ages, in a few cases the insertion of the metal studs to fix the strain gages introduced weakness along the gage fixation line and crack occurred at that line instead of occurring across the notch. Such irregular cases were discarded and the rest of the cases where fractures occurred through or near the notched section were considered only.

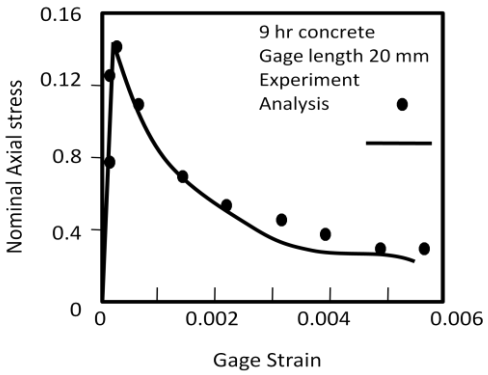

Fig. 7. stress-strain relation for 9 hrs concrete

Fig.7. to Fig.14. reveal that in all cases a fairly good agreement between experiment and numerical analysis is obtained. However, it should be noted that these results arc obtained using the data shown in Table 3 It is possible to alter the values of material parameters and to obtain different results, in that case the experimental stress-strain curves be different.

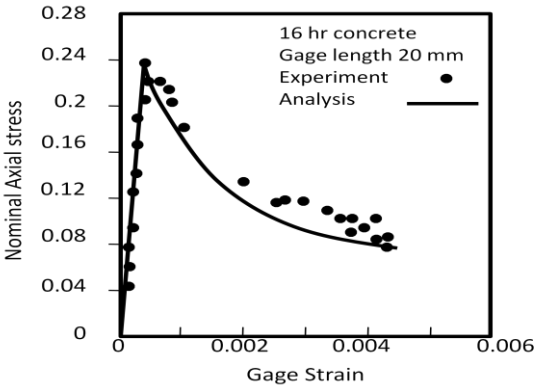

Fig. 8. stress-strain relation for 16 hrs concrete

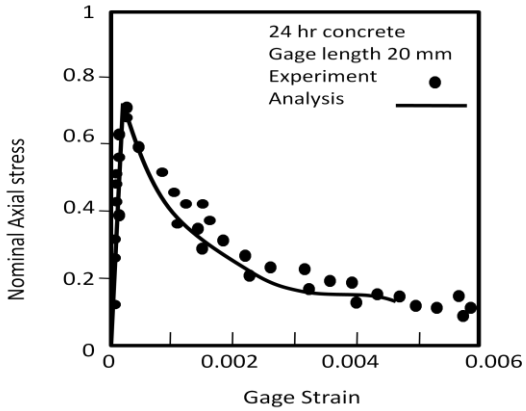

Fig. 9. stress-strain relation for 24 hrs concrete

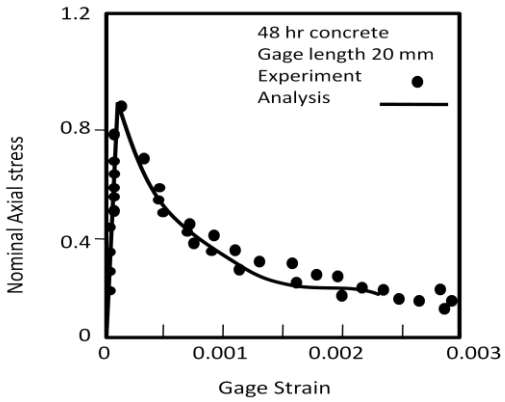

Fig. 10. stress-strain relation for 48 hrs concrete

Also it is possible to simulate the same experimental stress strain data using different combination of material parameters ku and / unless we have any data related to the localization process, its width etc. In this study we are dealing with localized fracture. Therefore some pertaining data on localization and fracture must be used to fix a unique set of material parameters.

Here it is done by matching the stress strain results obtained for three different gage lengths. Although we do not have any explicit data on width of fracture process zone, the three stress-strain relations corresponding to three different gage lengths comprise a unique set and when numerical results are matched with these, we obtain a unique set of material properties. Here only the result for 72hr concrete is shown in detail (Fig.11., Fig.13. and Fig.14.). These figures show fairly good agreement between test and analysis for all three cases of gage length. For brevity results for only one gage length in case of other gages, a kink in the stress-strain diagram for 9 hour concrete is observed when the strain is about 0.003. Similar case happened for 102 hour concrete also. This is due to the fact that at this point all the gauss integration points (points at which stress, strains, damage etc. are monitored) reach the ultimate value of equivalent fracture strain k_{u1}.

At this stage the value of softening parameter h depends only on g(K) and hence becomes positive at a few gauss points in the mostly stressed elements along the notch which is a violation of the gradient plasticity formulation. However, to maintain the non-local nature of the gradient plasticity approach g should not be allowed to become absolutely zero.

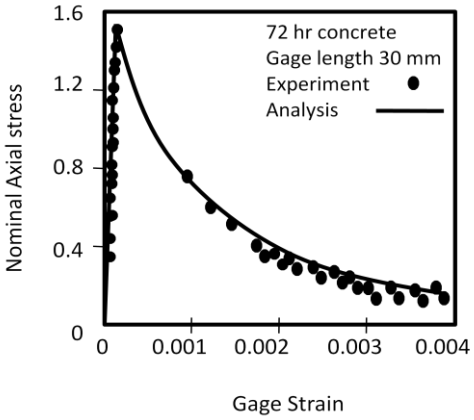

Fig. 11. stress-strain relation for 72 hrs concrete

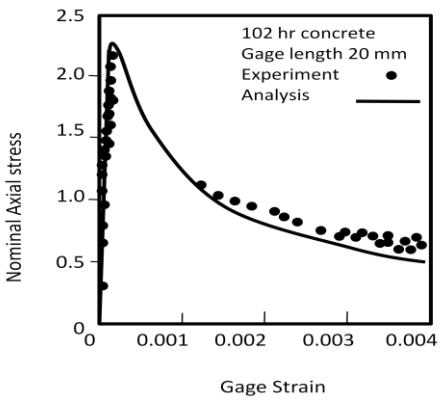

Fig. 12. stress-strain relation for 102 hrs concrete

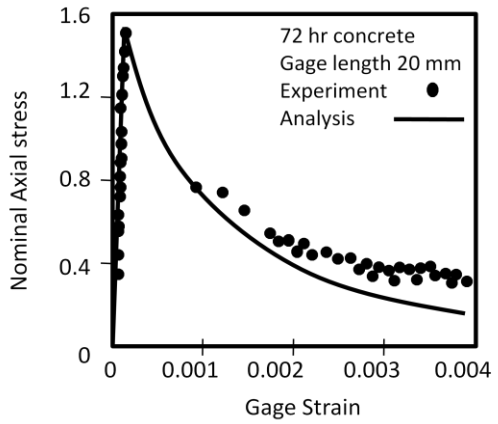

Fig. 13. stress-strain relation for 72 hrs concrete

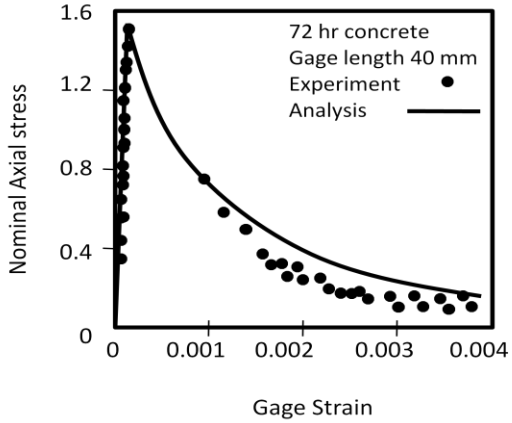

Fig. 14. stress-strain relation for 72 hrs concrete

4. CONCLUSION

The applicability of the gradient plasticity approach in the analysis of the fracture and strain localization phenomena of early age and reinforced concrete is investigated. The study enabled us to have a meaningful insight regarding the computational modeling of the fracture and localization of concrete especially at early age. The use of Modified Lattice Model proved to be successful in capturing strain localization phenomenon numerically.

Experimental tests are done under tension loading in servo control so that the full post-peak softening branch can be obtained experimentally. The numerical results using the Modified Lattice Model are compared with the experiment results. The main object of this study is the clarification of the fracture and localization problems of concrete at early age. The fracture energy of the early age concrete elements is calculated to capture the probability of cracking initiation. It is seen that if the fracture energy of the concrete element reaches to 0.92 MPa, the cracks start to be clear in the structural element. From the results shown in this research, it is proved that the Modified Lattice Model which is produced before by the author[9,10,11] is a suitable simulation model to capture the fracture energy and the strain localization of concrete elements at early ages.

From the results, it is proved that as the concrete gets matured it becomes more brittle and the width of the fracture process zone gets narrower because the brittleness increases. The relationship of l with age of concrete cannot be linear because in that case l will become zero at some later age. This is not practical because zero / means that the width of the fracture process zone will become zero.

The results also suggest that the gradient influence factor g needs to be modified at the later stage of loading (when $k \rightarrow k_u$) in order to render more realistic stress-strain relation at higher value of damage. In order to establish some definite relation of some model parameters like l and k_u with age of concrete or other physical properties of concrete more experimental study is necessary.

ACKNOWLEDGEMENT

The authors would like to acknowledge the head of deanship of scientific research at Najran University. This research is carried out and financially supported by Center of Scientific Research of Najran University, KSA with a serial number NU 33/11. Thanks are extended to the dean of faculty of Engineering of Najran University, for his grateful and continuous support.

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ظاهرة تركيز الانفعالات وحساب طاقة الانهيار للخرسانة الحديثة الصب

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الملخص:

في هذا البحث تم إجراء العديد من التجارب المعملية على الخرسانة حديثة الصب وفي أجواء المملكة العربية السعودية والمتميزة بالحرارة العالية نسبيا، وذلك مع الأخذ في الاعتبار الطاقة المفقودة نتيجة الاحتكاك للعينات أثناء عملية الصب، تم اختبار العينات الخرسانية الحديثة الصب تحت تأثير الشد المباشر وبطريقة مناسبة لحساب الطاقة اللازمة لانهيار العينة وذلك باختلاف زمن الصب. في هذا البحث تم أيضا التعرف على أماكن تركيز الانفعالات للعينات الخرسانية الحديثة الصب تحت تأثير الشد المباشر كما تم الوصول بالعينات لما بعد القيم العظمي للإجهاد، تم مقارنة النتائج المعملية مع النتائج النظرية والمستنتجة بطريقة النموذج الضلعي المعدل Modified Lattice Model والذي تم استنتاجه ونشره من قبل بمعرفة الباحث، وقد لوحظ التطابق الكبير بين النتائج المعملية والنتائج النظرية.